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Level I-A unit areas cover the meaning of "number," addition, multiplication, subtraction, and division. Level I-B covers introduction to common fractions, combining fractions, fractional parts, separating fractions, and applications.

Each unit under I-B has subdivisions, with a paragraph of teaching hints and examples. Stress is upon concrete examples, with terms introduced after understanding and without undue emphasis. The textbook may be used as a source for examples and problems after an idea has been grasped. Generally, textbook discussions are too abstract for use by Talent Preservation students.

Cookbooks, sewing instructions, and magazines such as Popular Science and Popular Mechanics are examples of good sources for practical work with fractions.

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O U T L I N E

of

MATHEMATICS — LEVELS I-A and I-B

for

Talent Preservation Classes

in

Grade Seven

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Houston Public Schools

September 1963

TPP MATHEMATICS — LEVEL I-A

Course Outline

I. Meaning of "number"
   - A. Number
   - B. Numeral
   - C. Decimal numeration
     1. Small numbers
     2. Large numbers
     3. Numbers larger than hundreds
   - D. Rounding numbers

II. Addition
   - A. Meaning of addition
   - B. Addition "facts"
   - C. Two-digit addition, two addends
   - D. Carrying
   - E. Three addends
     1. One-digit addends
     2. Two-digit addends
     3. Three-digit addends
   - F. Estimating
   - G. Six-addend addition
   - H. Larger addends

III. Multiplication
   - A. Meaning
   - B. Multiplication "facts"
   - C. Three-factor multiplication
   - D. Multiplication of larger numbers
     1. Estimating
     2. Two-digit numbers by one-digit multiplier
     3. Multiplying by a two-digit multiplier
     4. Three-digit multiplier
   - E. Factoring

IV. Subtraction
   - A. Meaning (opposite of addition)
   - B. Checking subtraction
   - C. Subtracting without borrowing
     1. Estimating

2. One-digit
3. Two-digit
4. Three-digit

D. Subtracting with borrowing

E. Comparing numbers by subtracting

V. Division

A. Meaning (opposite of multiplication)

B. Checking division

C. The remainder

D. Division by a one-digit divisor

1. One-digit dividend
2. Two-digit dividend
3. Estimating
4. Three-digit dividend

E. Division by larger divisors

F. Averages

G. Even numbers

TPP MATHEMATICS — LEVEL I-B

Course Outline

I. Introduction to Common Fractions

- A. The meaning of fractions
- B. Unit fractions
- C. Symbols for fractions
- D. Equivalent fractions
- E. Comparing fractions
- F. Rounding fractions

II. Combining Fractions

- A. Like fractions
- B. Adding fractions
- C. Multiplying fractions by whole numbers

III. Fractional Parts

- A. Fraction of a group
- B. Multiplying fractions
- C. Ratio
- D. Equivalent fractions, reducing

IV. Separating Fractions

- A. Subtraction
- B. Division

V. Applications

* * * * *

I. Introduction to Common Fractions

The concept of fractions seems to be the most difficult idea in arithmetic to get across in a Talent Preservation Program class. Therefore, spend as much time as necessary. Be sure that the basic ideas are well understood before going on. Be as concrete as possible in your presentation. Use paper, pieces of candy, pictures --whatever can be placed in the hands of the pupil--to clarify the idea of a fraction. Following the introduction of a topic, let the students work first with actual objects, then with diagrams, and finally with abstract numerals. The discussions presented in the textbook are generally too abstract for use by Talent Preservation students. After an idea has been grasped, however, the textbook may be used as a source of examples and problems for computation. The discussions in the textbook can also be used for reviews. Relate all work to concrete objects, but stress the estimation of answers, and adequate understanding of the processes being used. The mathematical terms generally used in this type of work are not stressed in this class. After an idea has been thouroughly digested by the class, the terms may be introduced, but without undue emphasis.

A. The meaning of fraction

Demonstrate that a fraction represents parts. Do not define a fraction as less than a whole. Cut up paper (or whatever will catch the interest of the class) to demonstrate different sizes of fractions, without actually naming them at this time. For example, you might cut one piece of paper into thirds; another, the same size but of a different color, into fourths; another, the same size but again of a different color, into sixths; and so on. Be sure to include both proper and improper fractions.

B. Unit fractions

Again, demonstrate fractions, but do not introduce the term unit fraction. Try to develop the concept of equal parts, along with an understanding that the number of equal parts in a whole determines the name of the fraction. Avoid using the word "whole" consistently, however. Stress the idea that a fraction is the result of a division.

C. Symbols for fractions

The students have already been introduced to one of the symbols for fractions. When they studied the process of division, they used the symbol $\frac{a}{b}$. Use of this same symbol to indicate a fraction may help to strengthen the idea that a fraction is a division. Use concrete objects again to demonstrate the meaning of the numbers in the numerator and the denominator. Consider the needs of the class in determining whether the terms numerator and denominator should be introduced at this time. Use examples of both proper and improper fractions in all work.

D. Equivalent fractions

In presenting the idea of equivalent fractions, keep the work entirely concrete at first. Let the students cut colored paper into different fraction sizes; for example, sheets of colored construction paper can be cut into halves, thirds, fourths, sixths, eighths, twelfths, and sixteenths, using paper of a different color for each unit. After the pieces have been used in demonstrating the various sizes of fractions, they can be reassembled into wholes, labeled, and mounted, leaving enough space between each piece to show the division. After enough work of this kind has been done, the symbols for these ideas, e.g. $\frac{2}{4} = \frac{1}{2}$, may be introduced. The presentation must still be done concretely, however, since no algorithm for finding equivalent fractions has been introduced. The sketching of simple diagrams such as the following provides one way of presenting and identifying equivalent fractions:

Introduce the idea that $\frac{2}{1} = 2$. This may be handled in several ways. Probably the least abstract approach is to point out that an object divided into 3 parts yields $\frac{1}{3}$'s, and an object divided into 1 part yields $\frac{1}{1}$'s. Since an object divided into 1 part is the object itself, $\frac{2}{1}$ means 2 parts, each part being a whole object; thus $\frac{2}{1} = 2$. It should be possible at this stage for the students to write some simple equivalents for both proper and improper fractions.

E. Comparing fractions

As the students begin to understand fractions, lead them into comparisons of fractional parts, such as: 2/3 is greater than 1/3; 1/4 is greater than 1/5. Although this probably will be obvious to most of the students, particularly if it is presented concretely, it should be pointed out specifically.

F. Rounding fractions

After the pupils have attained a good understanding of the relative size of fractions, discuss the rounding of fractions. A number line may be useful here; but the work should begin with concrete objects. Once the class is able to round fractions, all answers to examples and problems involving fractions should be estimated.

II. Combining Fractions

As usual, keep all work as concrete as possible, particularly when a new topic is being introduced.

A. Like fractions

Do not stress the name, <u>like</u> fractions, nor the term <u>common denominator</u>. Lead the class to see that there is no way to <u>name</u> the results of putting a half pie on the same plate with a fourth pie, unless we can think of both pieces in terms of the same units. Continue the presentation along this line, working only with fractions for which some object has been made. At this point, the class should have some understanding of 1/2's, 1/3's, 1/4's, 1/6's, 1/8's, 1/12's, and 1/16's. If so, now introduce 1/5's, 1/10's, and 1/9's. Through such demonstrations as cutting paper, drawing and dividing rectangles and circles, and perhaps even dividing candy bars or small cakes, lead the class to discover that the purpose of developing like fractions is to find a name for the result of an operation with fractions. For the present, limit the presentation to the use of diagrams and intuition for finding like fractions.

B. Adding fractions

Using concrete objects, introduce the adding of fractions. At first, limit the presentation to like fractions. Do not refer to "reducing" or "best form" at this point. Lead the class to discover, through manipulations involving fractions of real objects, that 1 half plus 1 half is 2 halves, just as 1 anything plus 1 anything is 2 anythings. After the class has worked with real objects for a time, progress to diagrams, then to abstract numerals.

Now go back to concrete objects and add unlike fractional parts. The students should be able to recognize, on the basis of their work with like fractions, that the sum of unlike fractions cannot be named. In other words, a half-piece of paper and a fourth-piece of paper may be stuck together with tape, but there is no name for the result in terms of a whole piece of paper. Through class discussion, lead the students to recognize that both pieces may be thought of in terms of fourths. Begin by adding unlike fractions where one of the fractions is already in terms of the common demoninator; for example, 1/2 plus 1/4; 1/3 plus 5/12. Progress from concrete objects to diagrams, then to numerals. Proceed slowly and make sure that each student is able to understand what he is doing. He can gain speed later. Particularly with this class, speed is not the primary object.

When each student thoroughly understands what is happening when fractions are added, introduce the idea of putting the sum into "best form." Point out that fractions are most easily understood when they are stated in the largest possible units. For example, 4/8 is the same as 2/4, and also 1/2. Since halves are the largest units, this is what we will call the fraction.

The changing of mixed numbers to pure fractions, and the changing of fractions to mixed numbers may also be introduced at this time. Through manipulation of concrete objects, lead the class to a sure understanding that eight eighths, or four fourths, is one. Although this may take longer than you had anticipated, be patient; do not give the class an algorithm for changing mixed numbers to fractions. Let the students develop this idea intuitively. Use concrete objects to introduce addition of mixed numbers; then progress to diagrams and later to numerals. Continuing in this manner, introduce the process of carrying in addition. Whenever the students encounter difficulty in understanding, review the process with concrete objects. Do not stress the use of specific terms. Although adding 1 3/4 to 2 1/8 is done the same way as adding 3/4 to 1/8, it doesn't sound the same if we call one "adding mixed numbers."

One common source of error when adding mixed numbers is the failure to add the whole numbers. To help in avoiding this oversight, require that the students write such additions as follows:

$$\begin{array}{rcl} 2\ 3/4 & = & 2\ 3/4 \\ +\ 1\ 1/2 & = & 1\ 2/4 \end{array}$$

Continue instruction and practice in the adding of fractions until the students understand it well. Use the textbook as a source for examples and problems, but screen them carefully, since the class has been exposed to only selected operations with fractions. As the work progresses, introduce other fractional units gradually, always with the use of concrete examples. Lead the class to see the relationships between such fractional units as thirds, fifths, fifteenths. Have the students draw diagrams to show that fifths, divided into thirds, yield fifteenths of the original whole, and other similar diagrams.

C. Multiplying fractions by whole numbers (multiple addition)

Through the use of concrete objects, lead the class to see that multiplying a fraction by a whole number can be effected by multiplying the numerator by the number. Do not confuse this kind of multiplying with the "1/2 of 4" type which will be discussed later. For the present, discuss only such things as four 1/2's, six 2/3's. Review frequently, and state all products in the largest possible units.

III. Fractional Parts of: a Group, a Fraction, a Whole

Do not introduce this unit as a study in multiplying; 1/2 <u>of</u> 4 is a very different idea from multiplying 1/2 <u>by</u> 4.

A. Fraction of a group

At this point, the class has not yet discussed fractions as parts of a group of objects. Spend as much time as necessary to develop this idea. A box of toothpicks can be used very effectively for this purpose. Starting first with easily divided groups, show how a group of toothpicks may be separated into equal parts. Use the word "of" when discussing the fractional parts. For example, have each pupil arrange 8 toothpicks neatly, like this: '''' ''''

Ask him to divide the group into 4 equal parts, then stop and name the parts, as follows: "One of four equal parts is one-fourth." Through manipulating various objects in this way, the student can discover for himself the meaning of 1/4 of 8, and other fractional combinations. (Small pieces of candy might make more "interesting" groups.)

After thorough practice and experience in finding fractional parts of concrete groups, proceed to the use of diagrams, then to numerals. Keep the work simple, using numbers that permit the drawing of simple diagrams to illustrate the problem at hand. Lead the class to discover the relationship between division and fractional parts. Then progress to less obvious divisions; for example, what is 1/3 of 8? Work a number of such problems, first using concrete objects for illustration, then diagrams. Lead the students to suggest dividing the group by the denominator of the fraction. Review again the relation of division to fractions.

Through manipulation of concrete objects, introduce the concept of more than one fractional part of a group; for example, 2/3 of 9. Have the students work in this way: first find how many toothpicks are in 1/3 of 9 toothpicks, then find how many are in two 1/3's of 9. Provide much practice in this process, first with concrete materials, then with numerals. In each case, be sure that the assigned group is evenly divisible by the denominator.

After enough written work has been done to determine that the class is ready, proceed to problems involving groups which are not evenly divisible. By this time the students should be able to state an algorithm for finding a fractional part of a number. Provide adequate time for the students to develop this understanding. If the students fully understand what is happening when 2/3 of 16 is found, the objective of this unit will be accomplished. It is not important that 2/3 of 16 be found quickly.

B. Multiplying fractions

Earlier, the relationship between certain fractional units (such as thirds of fifths are fifteenths) was discussed. Review this idea with the students preparatory to introducing fractional parts of fractions. Make extensive use of concrete objects and diagrams to help the students understand the meaning of 1/3 of 1/5: that the answer, 1/15, refers to its value as part of the original whole. Lead the class to discover that the new fractional unit is the product of two old units, since there are 3 thirds in 1 fifth, and 5 times 3 thirds in 5 fifths. When this is adequately understood by the students, proceed, through the use of concrete objects, diagrams, and numerals, to an understanding of 1/3 of 2 fifths, then 2 thirds of 1/5, and finally 2/3 of 2/5.

Since the students have already learned how to change mixed numbers to fractions, introduce them, after some review, to the multiplication of mixed numbers. Then lead them to arrive at an algorithm for multiplying fractions. When this process is clearly understood by the class, provide for practice, using examples from the textbook, along with other problems drawn from newspaper articles, cookbooks, and similar sources. Discuss multiplication processes with more than two factors, and review the class regularly, through the use of diagrams and concrete objects on the meaning of such examples as 2/3 of 3/4.

C. Ratio

Use concrete objects to present the idea of "what-part-of." Limit the presentation to simple numbers, so that the class can see, using toothpicks or other small objects, that 4 is 4/8 of 8, and 3 is 3/5 of 5. With larger groups of objects, approach the subject in this manner: first, determine what part 1 toothpick is of the total group (for example 25 toothpicks). Since 7 toothpicks are 7 times as much as 1 toothpick, 7 toothpicks must be 7 times that part. After the students have worked several problems with concrete objects, progress to diagrams, then to numerals.

D. Equivalent fractions, reducing fractions

The class has already used the idea of equivalent fractions intuitively. If the students now thoroughly understand the principle of multiplying fractions, an algorithm can be derived, as follows:

Example: 2/3 is a number.
A number x 1 is itself.
4/4 is 1.
2/3 x 4/4 then must be the same as 2/3.
2/3 x 4/4 = 8/12.
8/12 is the same as 2/3.

Have the class check such examples with objects or diagrams, so that they can relate the method or procedure to what is actually being done. After this basic idea is clear, the class can progress to the idea of rewriting a fraction as an equivalent fraction with any desired fractional unit. The same idea can be used to "reduce" fractions. The class was introduced to factoring in Level I-A. After reviewing this process, lead the class to see that 15/20 can be written as $\frac{3 \times 5}{4 \times 5}$, or 3/4 x 5/5, or 3/4 x 1, or 3/4. At the beginning, present all such steps through the use of diagrams.

IV. Separating fractions

A. Subtraction

Present subtraction of fractions in a manner similar to that for addition.

B. Division

Since division has been related repeatedly to fractional parts, division of a fraction should present no problems at this point. It was discussed in the "fractional parts of" section. When it is reviewed, lead the class to see that a number divided by 2 means the same as 1/2 <u>of</u> the number. Division by a fraction, however, often presents difficulties, mainly because one meaning of division, multiple subtraction, is not clear. Review the meaning of the statement, 12 ÷ 3 = 4, as saying that 12 contains four 3's. Thus, if we ask what is $6 \div \frac{1}{2}$, we mean, 6 contains how many $\frac{1}{2}$'s? Through the use of objects and diagrams, simple division examples of this type may be solved by counting. A number line is also a convenient way to present this topic.

When the class has begun to have a feel for the number of times a fraction is contained in a whole number, progress to examples in which mixed numbers divided by fractions and fractions divided by smaller fractions. A less obvious division such as 2/3 ÷ 7/8 should not be attempted at this time. The <u>idea</u> of division is the objective at present.

Division, as repeated subtraction, may then be presented. 3 ÷ 3/4 would be handled by subtracting 3/4 from 3 as many times as possible, then counting the number of times the subtraction was done. Lead the class to see (or maybe even suggest) the idea of division by the common denominator method. Refer constantly to physical objects.

V. Applications

As the class learns to use fractions, make use of the textbook, newspapers, magazines, books from other classes, and other materials as sources for work with fractions. Cookbooks, sewing instructions, and magazines such as Popular Science and Popular Mechanics are other examples of good sources for practical work with fractions.